# THE POWER PROCUMENT PROBLEM WITH POBABILISTIC CONSTRAINTS

MATTHIAS ONDRA

TÖK

TECHNISCHE UNIVERSITÄT WIEN
Vienna University of Technology

Institut für Managementwissenschaften

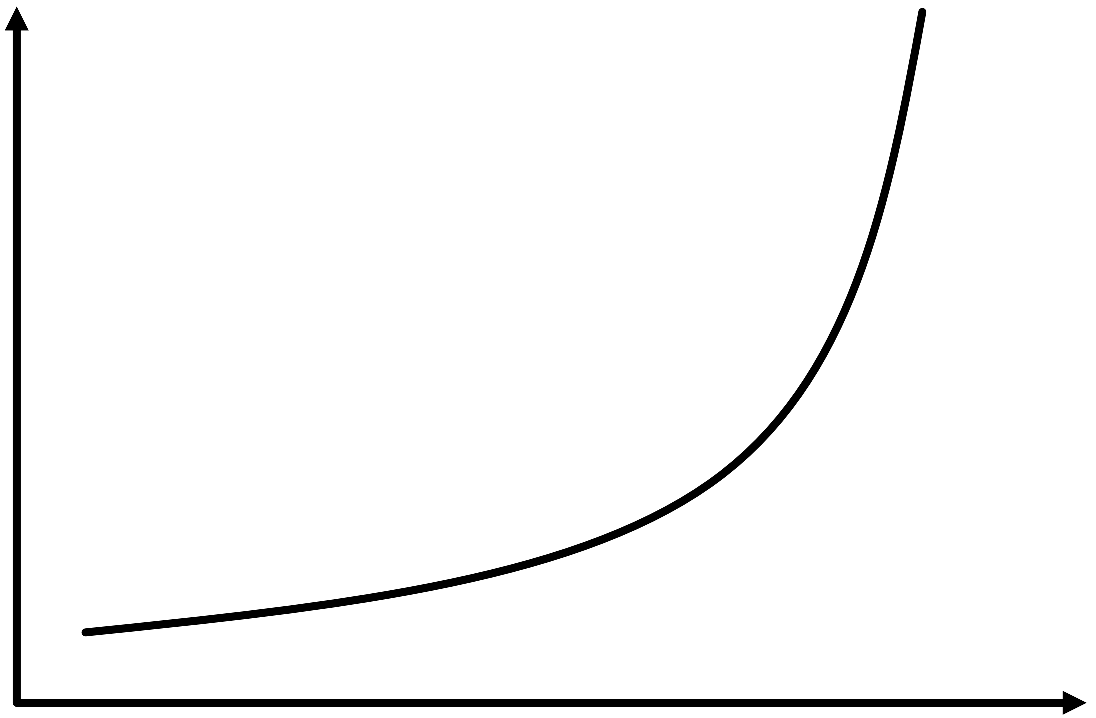
# MOTIVATION

- EU climate strategy: 2020, 2030, 2050: increase share in renewables: 20%, 32%, 75%; increasingly volatile power output
- Enterprises can act as prosumers (= consumer + producer who can cover a part of his demand)
- A prosumer acting in the liberalized power market faces the *power procurement problem* (PPP)
- (PPP): Procuring the required power at minimum possible costs (Rezaeipour and Zahedi, 2017), *[...] choose best strategy by considering available sources* (Shafieezadeh at al., 2019)

# OVERVIEW

- Generation expansion problem (GEP) from a prosumer's point of view (How much to invest in which technology? Energy park stand-alone application)
- Stochastic, data-driven modeling approach
  1. Probabilistic constraints: Supply-demand constraint holds true with an ex-ante chosen level of reliability
  2. Empirical data, stochastic approaches to the GEP assume stochastic variables to be Gaussian (Gen and Xi, 2019)

# OVERVIEW- EFFICIENT INVESTMENT FRONTIER




stand-alone application

$\chi$ exogenous

- Recover efficient frontier of optimal level of investment and reliability

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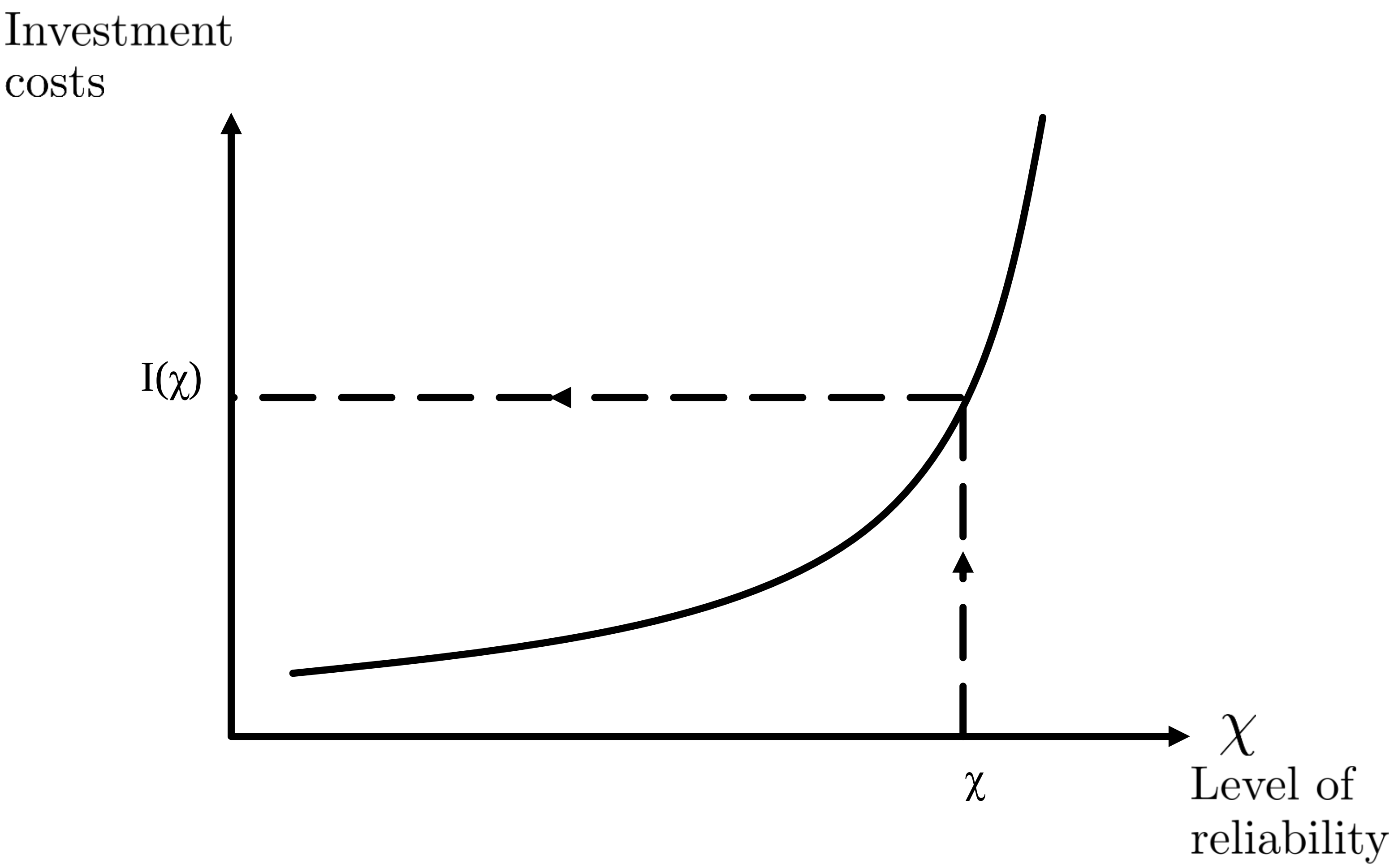


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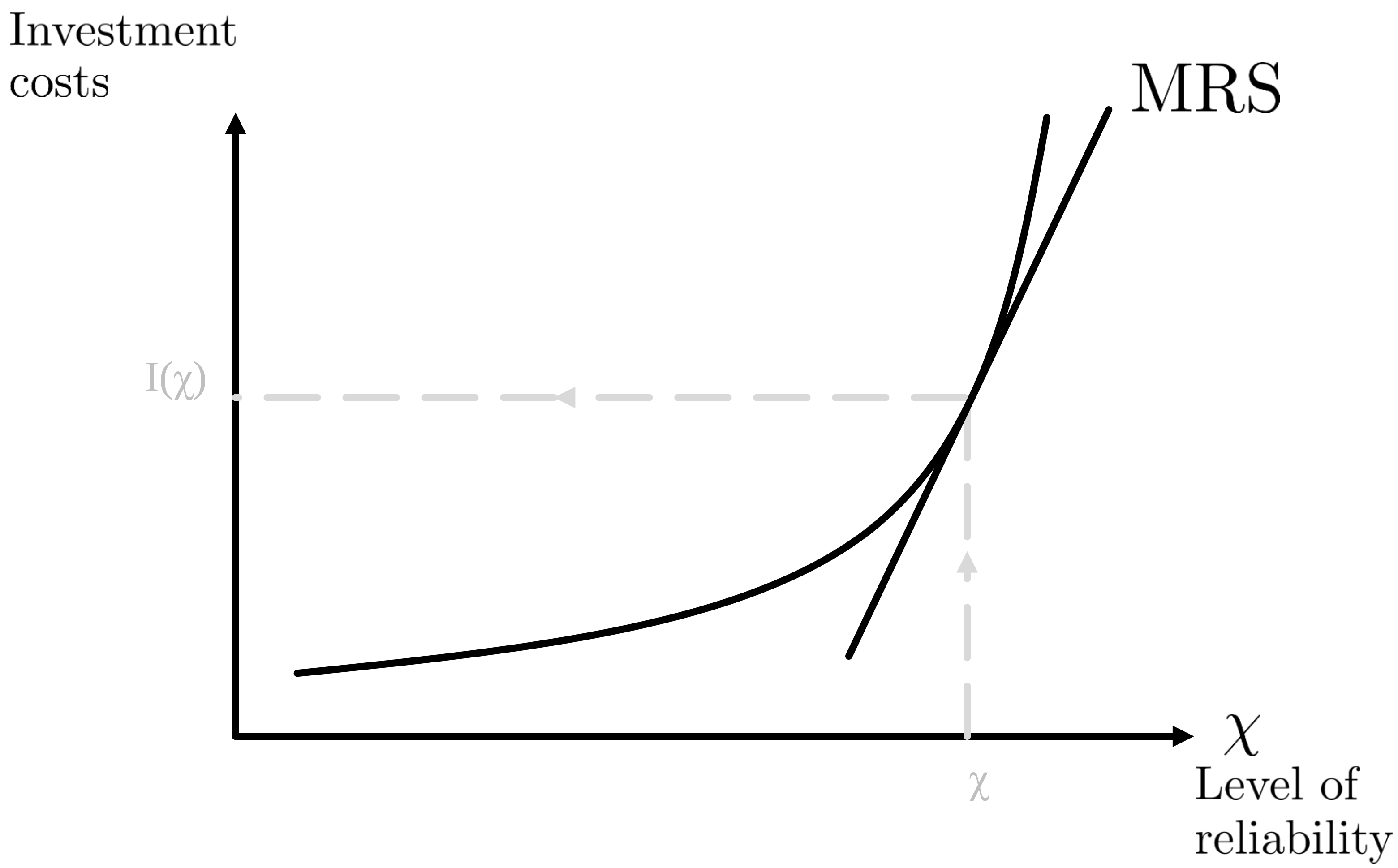
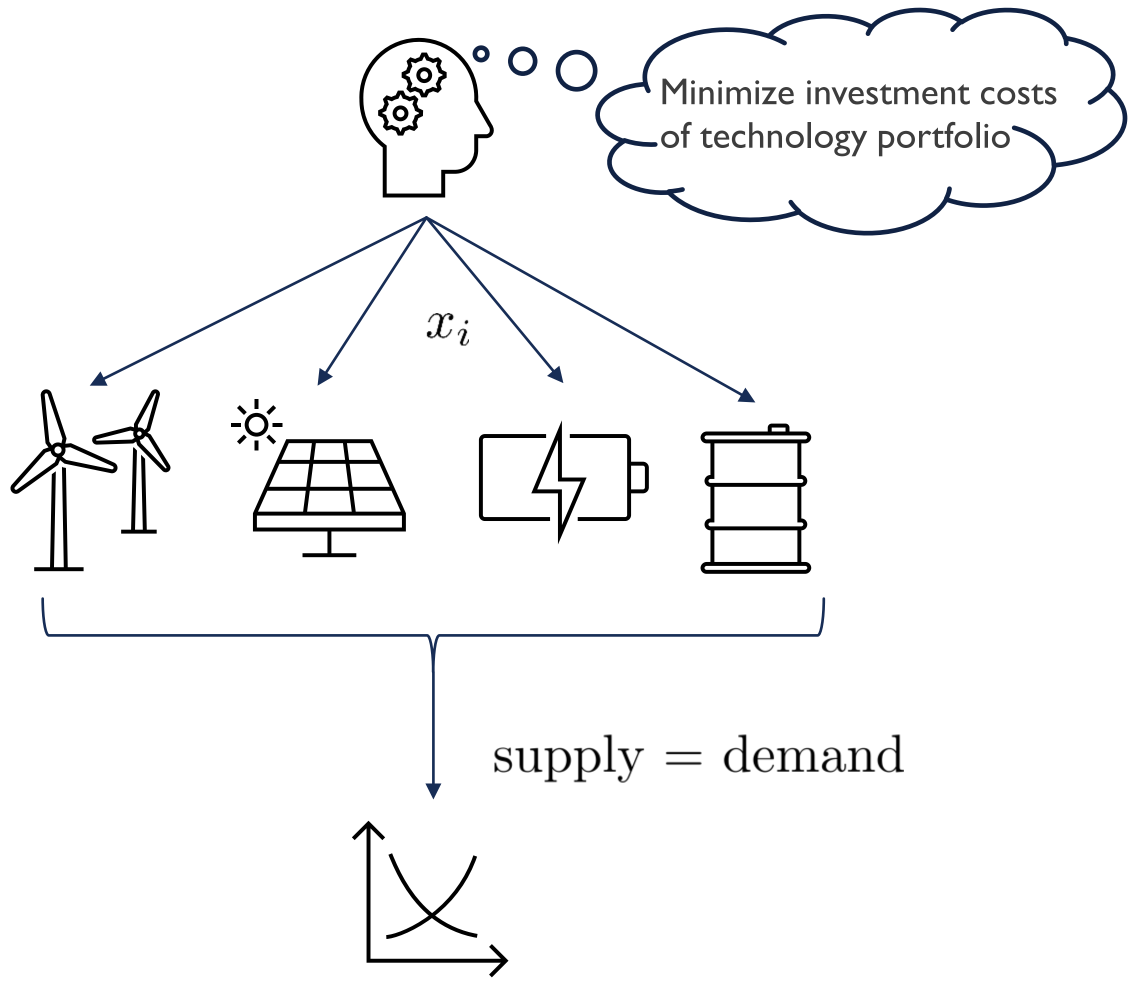



stand-alone application

$\chi$ exogenous

- Recover efficient frontier of optimal level of investment and reliability
- Derive the marginal rate of substitution between investment costs and reliability

# MODEL CONSTRUCTION
## DETERMINISTIC SUPPLY DEMAND CONSTRAINT



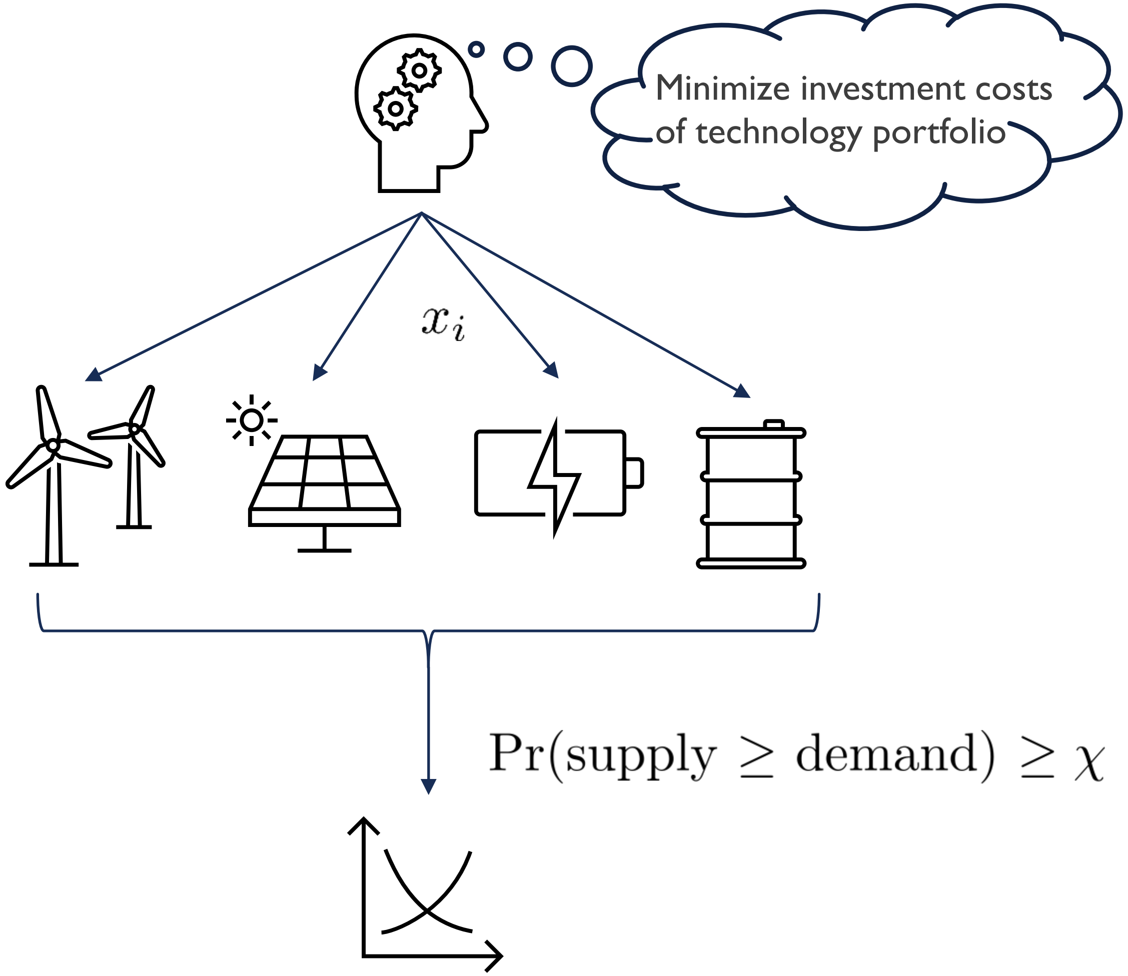

$$\min_{x_i \geq 0}\ x_1 p_1 + \ldots + x_n p_n \qquad \text{subject to}$$

$$x_1 P_{1t} + \ldots + x_n P_{nt} \geq d_t$$

| | |
|---|---|
| $x_i$ | capacity of i-th technology |
| $p_i$ | price per installed capacity of i-th technology |
| $P_{it}$ | power per installed capacity of i-th technology available at time $t$ |
| $d_t$ | demand at time $t$ |

# MODEL CONSTRUCTION

## PROBABILISTIC SUPPLY DEMAND CONSTRAINT




$$\min_{x_i \geq 0} \; x_1 p_1 + \ldots + x_n p_n \qquad \text{subject to}$$

$$\Pr\{x_1 P_{1t} + \ldots + x_n P_{nt} \geq d_t\} \geq \chi$$

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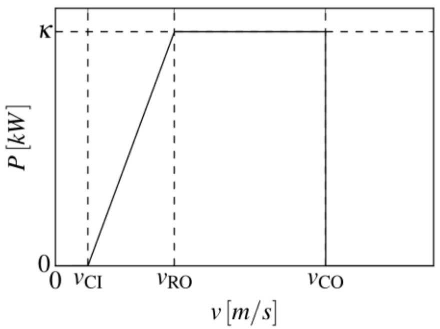
| $\chi$ | ex-ante chosen level of reliability |

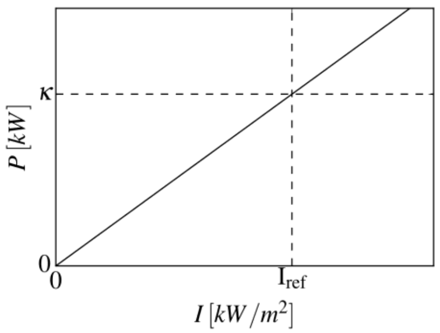
# MODEL CALIBRATION
## THE USE CASE

- Cost minimizing technology portfolio (wind and solar, n=2) s.t energy park (wind/solar technology) can supply hourly demand $d_t$ =100 kW in the daytime with probability $\chi$
- Wind: $p_1$= 1400 €/kW; Solar: $p_2$= 1000€/kW (Carlsson et al., 2014)
- Sample from real world data of solar irradiance and wind speed in Schwechat, Austria (hourly data available from 2012-2018)
- Wind speed and solar irradiance are translated into power output via the physical energy model

$v_t$ →




$I_t$ →



# MODEL SOLUTION

Replace probabilistic constraint by sample of N observations

$$\Pr\{x_1 P_{1t} + x_2 P_{2t} \geq d_t\} \geq \chi \quad \Longleftrightarrow \quad x_1 P_{1t}^{(i)} + x_2 P_{2t}^{(i)} \geq d_t^{(i)}, \qquad i = 1, \ldots, N$$

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## Sample Average Aproximation (SAA)

- Idea first introduced in Sen (1992); Formulation as a mixed integer program (Ruszczynski, 2002; Luedtke and Ahmed, 2008)
- Remove (1-$\chi$) % of scenorios; select responsive/ non-responsive scenarios via binary variable $z_m$
- Formulation as a mixed integer program; big M formulation

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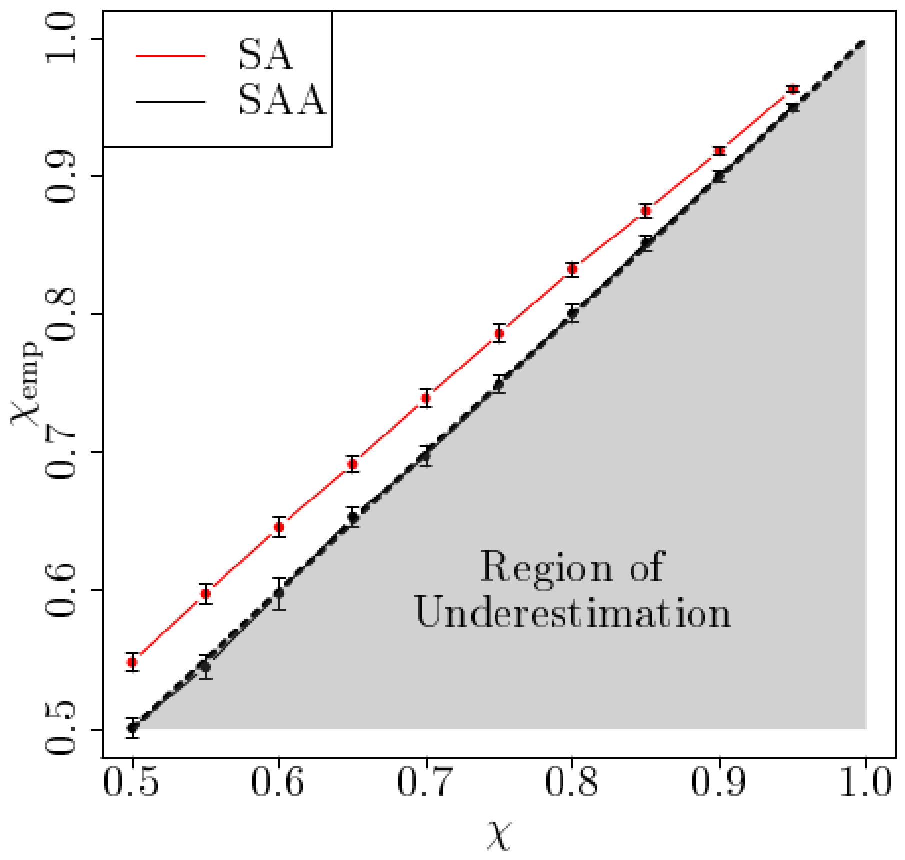
## Sample Approach (SA)

- Introduced in Calafiore and Campi (2005)
- Sample&Discard (Campi and Garatti, 2011): Remove (reliability dependent) number of constraints according to any algorithm
- Solution is robustly feasible with reliability level $\chi$

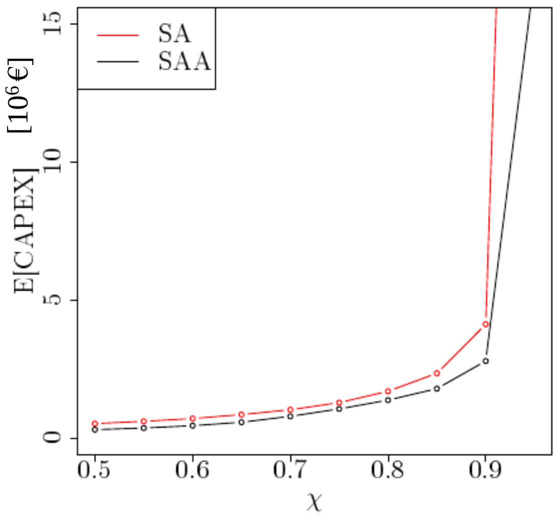
# MODEL VALIDATION

- Ex-post validation, resampled scenarios
- SA is a suboptimal routine and overestimates SAA
- SA introduces contingency capacities which refer to higher ex-post reliability levels

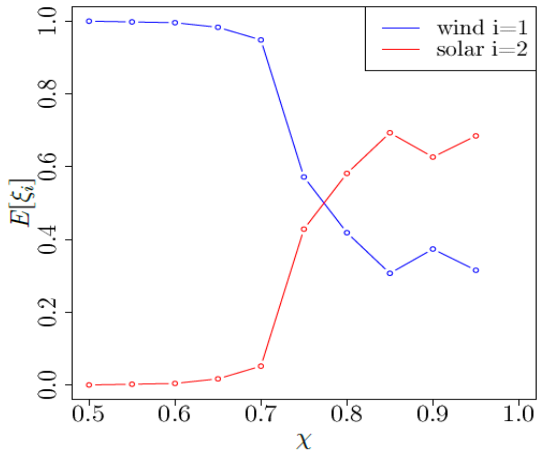


# MODEL ANALYSIS

- Recover energy manager's optimal level of investment given a reliability level
- Expected substitution rate between reliability and investment costs
- Over wide range of $\chi$ comes at a constant price; for high $\chi$ frontier is strictly convex
- Increased share in solar technologies with increasing levels of reliability




SAA



# OVERVIEW

- Extend the setup to on-the-grid scenario without feed-in-tariff
- In case of a power shortfall energy manager purchases residual power at the balancing market
- Probabilistically constrianed approch considers only the frequency of scenarios exceeding the probabilistic constraint but not the extent of violation
- What is the energy manager‘s optimal technology portfolio in case of (i) pre-contracted power and (ii) purchasing power at the spot market?

# MODEL CONSTRUCTION

- Total costs are given by
  1. Investment costs $I(\mathbf{x}) = \mathbf{p}'\mathbf{x}$
  2. Excess payments for purchasing power at the balancing market which occures
     in case of a short supply

$$X(\mathbf{x}) = d - \mathbf{x}'\mathbf{P} > 0$$

over the investment period $\Delta T$ at price $\xi$

$$\Delta(\mathbf{x}) = E[\Delta T \xi \max\{X(\mathbf{x}), 0\}]$$

- What is the optimal level of investment in RES when the outside option exists?

$$\min_{\substack{\mathbf{x},\\ z_1,\dots,z_N}} \quad \mathbf{p}'\mathbf{x} + \frac{\Delta T}{N} \sum_{i=1}^{N} z_i$$

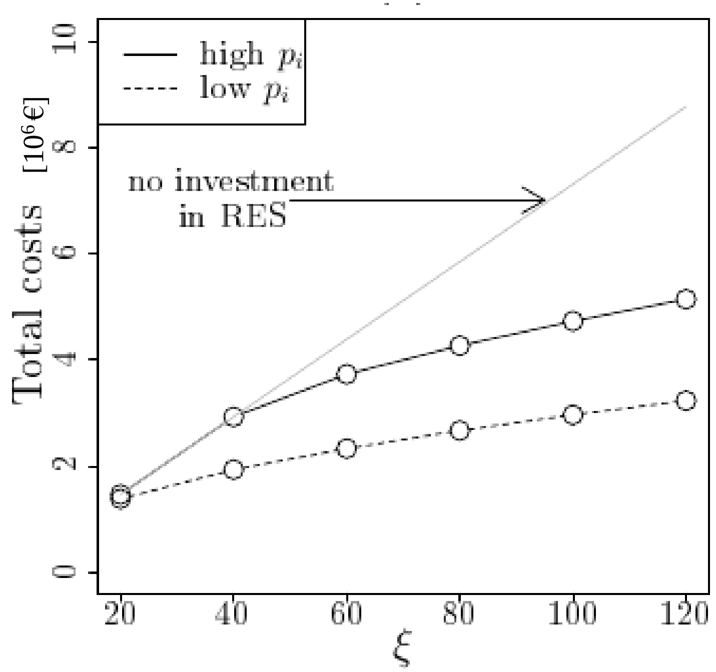
$$z_i \geq \xi_t^{(i)} (d_t - \mathbf{x}'\mathbf{P}_t^{(i)})$$

$$z_i \geq 0, \quad i = 1, \dots, N,$$

$$x_j \geq 0, \quad j = 1, \dots, n.$$

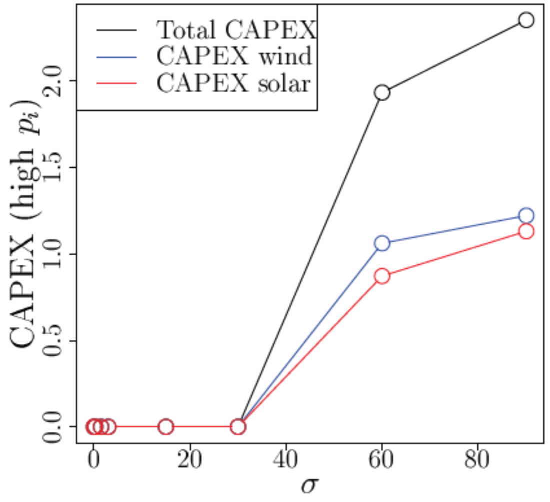
# RESULTS (1/2)

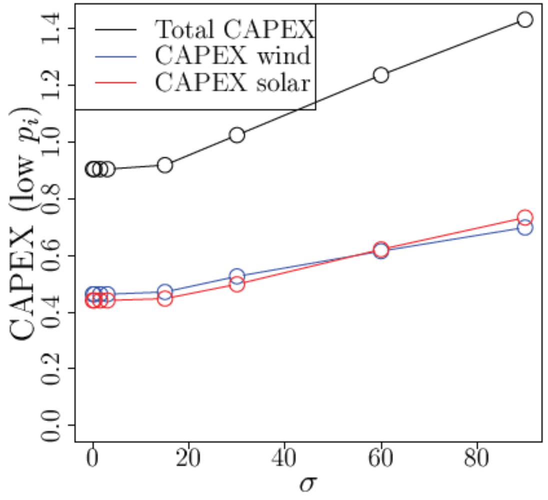
- Consider two scenarios associated with the prices of the investment goods
  - High prices of the investment goods (high $p_i$)
  - Low prices of the investment goods (low $p_i$)
- Consider two scenarios associated with the outside option
  - Purchase pre-contracted power with deterministic price $\xi$
  - Purchase power at the spot market with stochastic price $\xi \sim \mathrm{TN}(\mu, \sigma)$



# RESULTS (2/2)

- Consider two scenarios associated with the prices of the investment goods
  - High prices of the investment goods (high $p_i$)
  - Low prices of the investment goods (low $p_i$)
- Consider two scenarios associated with the outside option
  - Purchase pre-contracted power with deterministic price $\xi$
  - Purchase power at the spot market with stochastic price $\xi \sim \mathrm{TN}(\mu, \sigma)$

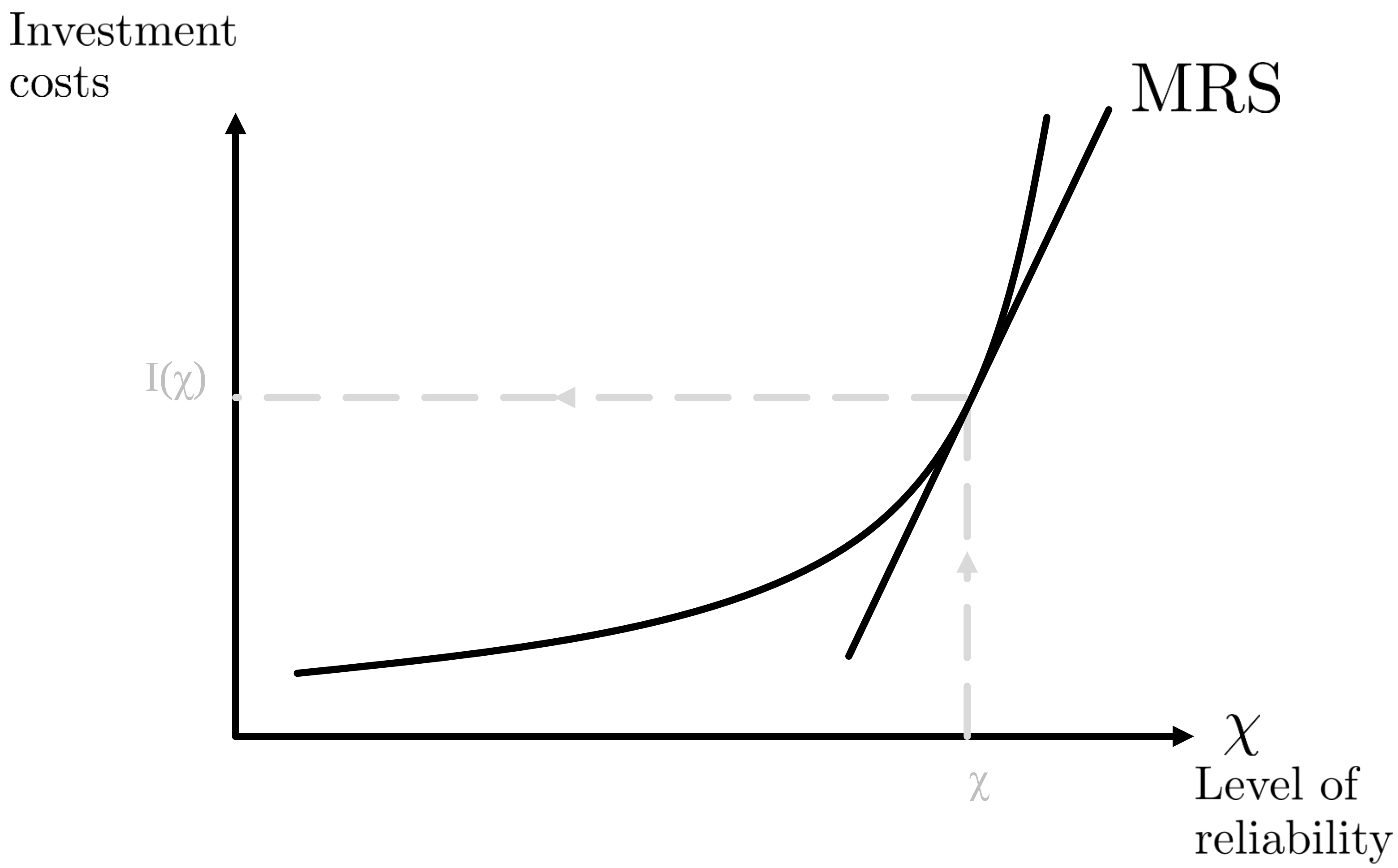


# OVERVIEW

- Consider again on-the-grid scenario without feed-in-tariff
- Energy manager includes option to procure power at the balancing market
- How does the value of the outside option influence the optimal level of reliability chosen by the energy manager?

# OVERVIEW- EFFICIENT INVESTMENT FRONTIER

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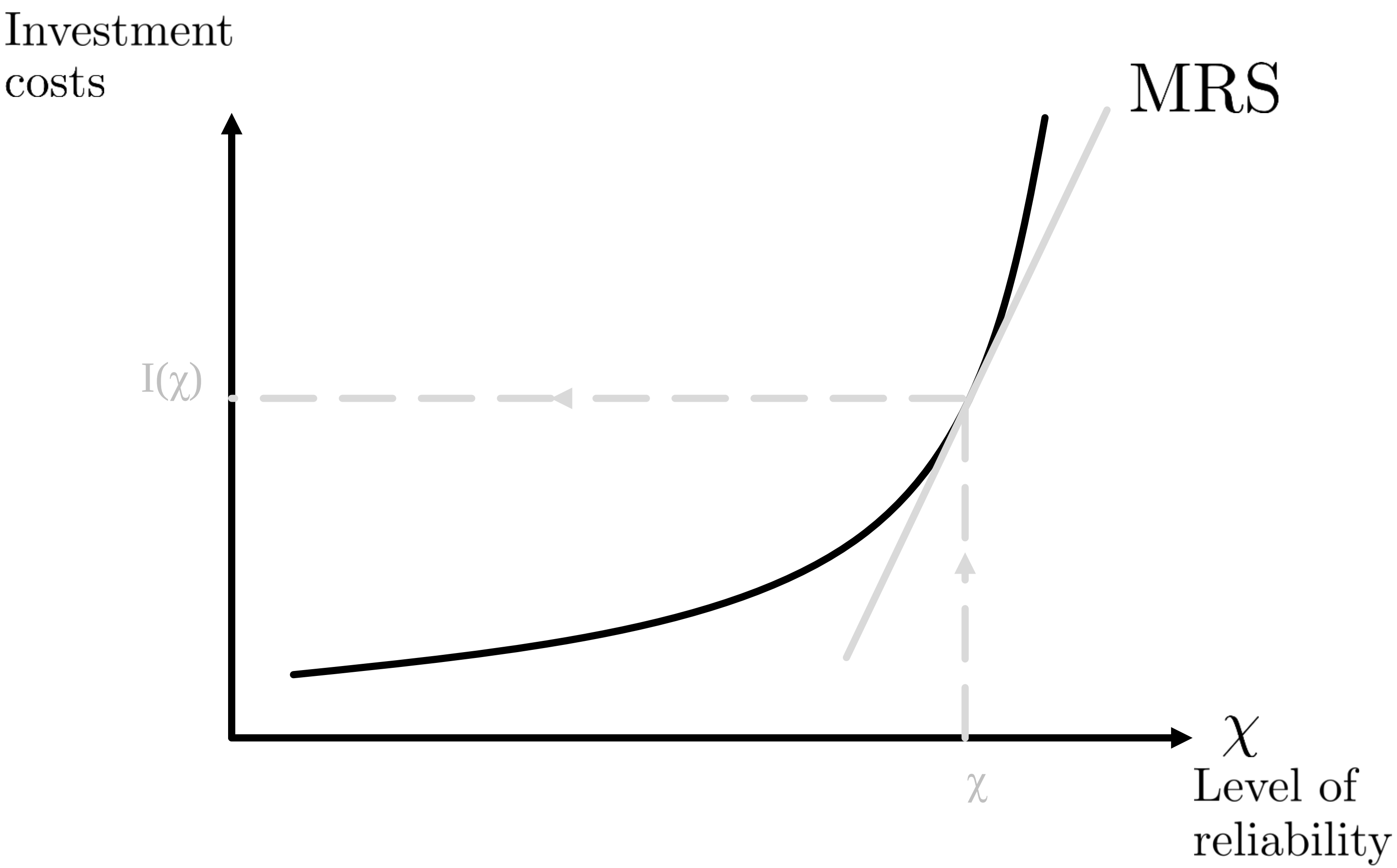
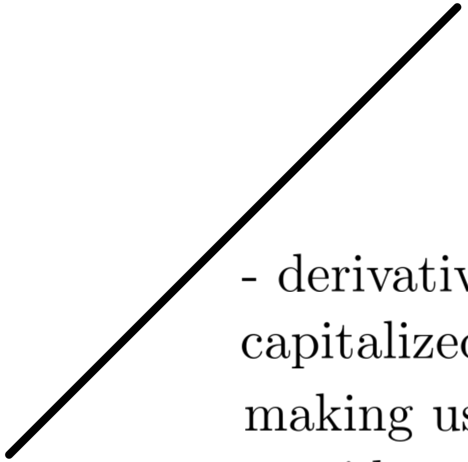
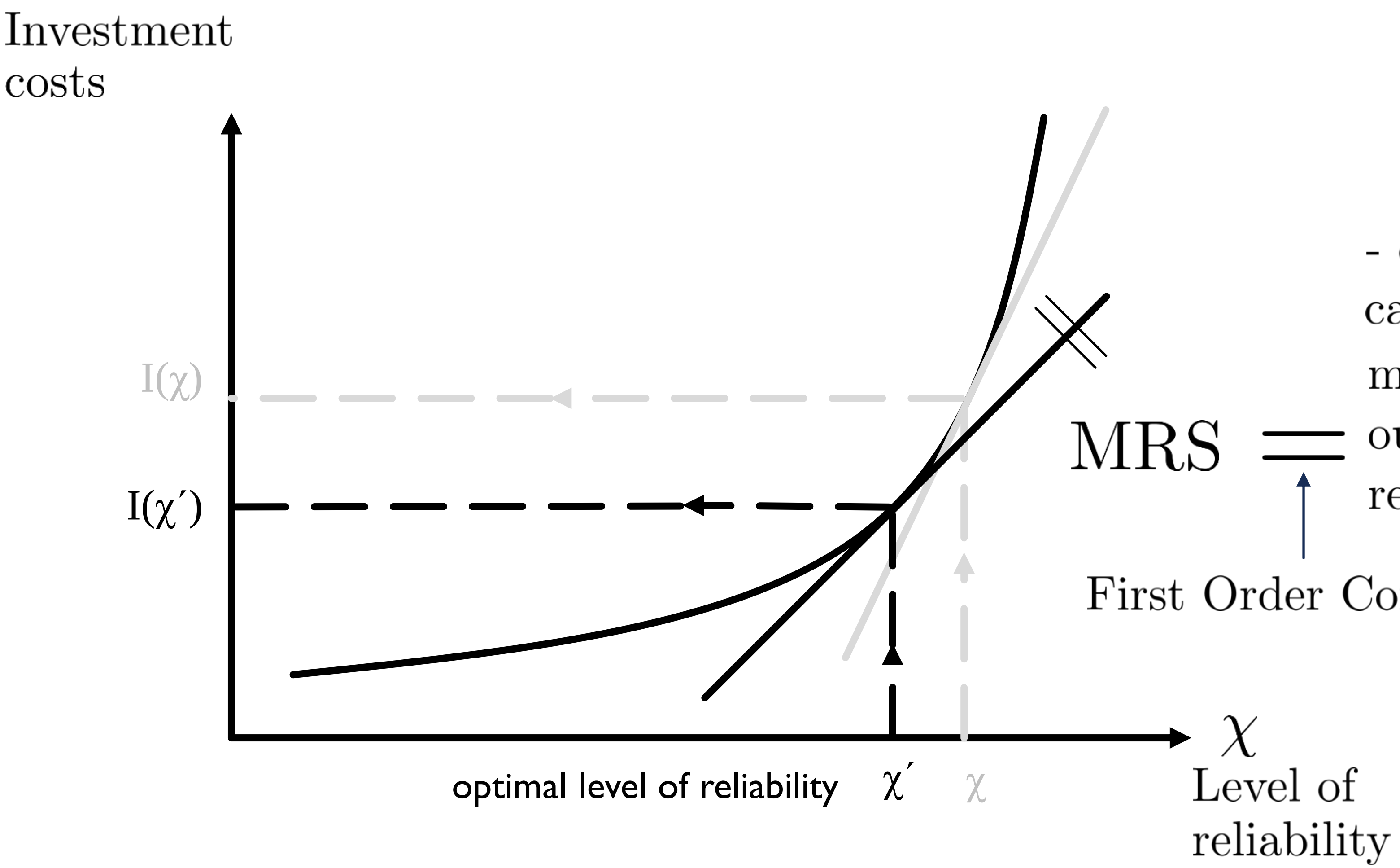



on the grid scenario
$\chi$ endogenous



- derivative of capitalized costs for making use of an outside option with respect to $\chi$

# OVERVIEW- EFFICIENT INVESTMENT FRONTIER




on the grid scenario
$\chi$ endogenous

MRS = - derivative of capitalized costs for making use of an outside option with respect to $\chi$

↑ First Order Condition

# INVESTMENT COSTS

- The energy park's level of reliability is $\chi = \Pr\{X(\mathbf{x}) \leq 0\}$
- The energy manager's optimal portfolio for an ex-ante specified level of reliability $\chi$ is given by $\mathbf{x}(\chi)$, i.e. the solution of the constrained cost-minimization problem

$$\min_{\mathbf{x}} \mathbf{p}'\mathbf{x}$$

$$\mathrm{VaR}_{\chi}(\mathbf{x}) = 0$$

$$x_i \geq 0, \qquad i = 1, \ldots, n$$

- Investment costs of the optimal portfolio are given by $I(\mathbf{x}(\chi)) = \sum_i p_i x_i(\chi)$

# EXCESS PAYMENTS

- In case of a short power supply, excess paymants are given by $\Delta(\mathbf{x}) = E[\Delta T \xi \max\{X(\mathbf{x}), 0\}]$
- Via total law of probabilities, reformulate

$$\Delta(\chi) = \Delta T E[\xi](1-\chi)\mathrm{CVaR}_{\chi}(\mathbf{x}(\chi))$$

- The total costs as a function of reliability are given by $C(\chi) = I(\chi) + \Delta(\chi)$
- FOC:

$$I'(\chi) = -\Delta'(\chi)$$

# TOTAL COSTS

- In case of a short power supply, excess paymants are given by $\Delta(\mathbf{x}) = E[\Delta T \xi \max\{X(\mathbf{x}), 0\}]$
- Via total law of probabilities, reformulate

$$\Delta(\chi) = \Delta T E[\xi](1-\chi)\mathrm{CVaR}_{\chi}(\mathbf{x}(\chi))$$

- The total costs as a function of reliability are given by $C(\chi) = I(\chi) + \Delta(\chi)$
- FOC:

$$I'(\chi) = -\Delta'(\chi)$$

$$\mathrm{MRS}(\chi) = \sum_i p_i \frac{\mathrm{d}x_i}{\mathrm{d}\chi}$$

?

# OPTIMAL LEVEL OF RELIABILITY

$$
\begin{aligned}
\frac{1}{\Delta TE[\xi]}\frac{\mathrm{d}\Delta}{\mathrm{d}\chi} &= \frac{\mathrm{d}}{\mathrm{d}\chi}\left((1-\chi)\mathrm{CVaR}_\chi(\mathbf{x}(\chi))\right) \\
&= -\mathrm{CVaR}_\chi(\mathbf{x}(\chi)) + (1-\chi)\left(\frac{\partial \mathrm{CVaR}_\chi(\mathbf{x}(\chi))}{\partial x_i}\frac{\mathrm{d}x_i}{\mathrm{d}\chi} + \frac{\partial \mathrm{CVaR}_\chi(\mathbf{x}(\chi))}{\partial \chi}\right) \\
&= (1-\chi)\frac{\partial \mathrm{CVaR}_\chi(\mathbf{x}(\chi))}{\partial x_i}\frac{\mathrm{d}x_i}{\mathrm{d}\chi} + \frac{\partial}{\partial \chi}\left[(1-\chi)\mathrm{CVaR}_\chi(\mathbf{x}(\chi))\right] \\
&\vdots
\end{aligned}
$$

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&= (1-\chi)\frac{\partial \mathrm{CVaR}_\chi(\mathbf{x}(\chi))}{\partial x_i}\frac{\mathrm{d}x_i}{\mathrm{d}\chi} + \frac{\partial}{\partial \chi}\left[(1-\chi)\mathrm{CVaR}_\chi(\mathbf{x}(\chi))\right] \\
&\vdots
\end{aligned}
$$

⇒ FOC: $\sum_i p_i \frac{\mathrm{d}x_i(\chi)}{\mathrm{d}\chi} = \sum_i \Delta TE[\xi] E[P_i \cdot \mathbf{1}_{X(\mathbf{x}(\chi))\geq 0}] \frac{\mathrm{d}x_i(\chi)}{\mathrm{d}\chi}$

# OUTLOOK

- How does correlated spot market price effect the energy managers optimal level of reliability?
- Regulatory aspects: How to incentivze the energy manager (by setting prices of the investment goods $p_i$) to obtain a specific optimal level of reliability?
- How do energy managers decide on the optimal level of reliability in a general equilibrium model?

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